Distribution of boulders and the gravity potential on asteroid Itokawa

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A R T I C L E   I N F O

Article history:
Received 18 December 2013
Revised 3 October 2014
Accepted 7 October 2014
Available online 22 October 2014

Keywords:
Asteroid Itokawa
Asteroids, surfaces
Impact processes
Geological processes

A B S T R A C T

The small asteroid Itokawa was visited in 2005 by the Japanese spacecraft Hayabusa. The images of the surface showed a scenario different to previously visited asteroids. Itokawa has a small number of large craters and many large boulders randomly distributed on most of the surface.

We analyse images taken at different surface resolution and configurations corresponding to several regions on the asteroid’s surface. By overlapping visual images and maps of the total potential and surface gravity, we observe a correlation between the distribution of boulders and these parameters. The boulders on the surface were identified by visual inspection of several images. After fitting ellipses to every boulder, we computed their size and the size distribution from decimeters to several meters at different locations in the surface. We found that the size distribution is correlated with the total potential and the surface gravity. A steeper size distribution shifted towards the small objects is observed in the low negative total potential (high surface gravity), which corresponds to the Muses-C region. Meanwhile, in the “head” and “bottom” regions of high potential (low surface gravity), we obtain a shallower size distribution, shifted towards the large boulders.

We confirm there is a size segregation that is correlated with the gravity field which can be explained under the action of the Brazil nut effect. There is a global relocation of boulders, with large ones going into the high potential regions and small ones into the low potential ones. A shape segregation is also observed on the location of the boulders: more rounded ones are found in the regions of high potential, while more elongated ones are frequent in regions of low potential.

1. The asteroid

In late 2005 the Japanese mission Hayabusa reached the Near-Earth Asteroid 25143 Itokawa (1998 SF36) (Fujiiwara et al., 2006). In the following months the spacecraft made a rendezvous around the asteroid acquiring images of its surface and selecting the sampling site where it made a couple of touch downs.

The images of the surface taken with the on board instrument AMICA showed a completely different picture in comparison to the other asteroids previously visited by spacecrafts: while the other asteroids (much larger than Itokawa) have a surface with many craters and generally covered by a thick regolith layer, Itokawa has a small number of large craters and, more strikingly, the presence of a large number of boulders randomly distributed in most of the surface (Saito et al., 2006).

Itokawa is an Earth-crossing asteroid with a perihelion distance of $q = 0.953$ AU and an aphelion distance of $Q = 1.695$ AU. The eccentricity of the orbit is moderate ($e = 0.280$), but its inclination with respect to the ecliptic is very low ($i = 1.622^\circ$). The asteroid is an slow rotator with a rotational period of 12.1 h (Fujiiwara et al., 2006) (similar reference for the following data). The asteroid’s dimensions are 535, 294, and 209 m ($\pm 1$ m). The total surface area is $3.93 \times 10^5$ m$^2$ and the volume is $1.84(\pm 0.092) \times 10^7$ m$^3$. The diameter of a sphere of equal volume would be 320 m. Itokawa is an S-type asteroid. The bulk density is $\sim 1.95(0.14)$ g/cm$^3$ (Abe et al., 2006).

The dynamical evolution of Itokawa at present is very chaotic due to frequent close encounters with the Earth. The low-inclination leads to a high impact probability with our planet. The question of dynamical origin was analysed by Michel and Yoshikawa (2005) in a statistical way, as it is not possible to trace the evolution from the source region to the present due to the chaotic nature of the dynamics. Using the NEO model of Bottke et al. (2002) and Michel and Yoshikawa (2005) determined the most likely source region for the present orbit of Itokawa: the $\nu_6$ secular resonance in the inner main belt (64% probability) and the Mars-croasser population (35% probability). This result is consistent with the S-type spectra of Itokawa.

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http://dx.doi.org/10.1016/j.icarus.2014.10.011
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It is worth to compare the small size of Itokawa, with the other asteroids visited by spacecraft: (951) Gaspra \( - 18 \times 11 \times 9 \) km, (243) Ida \( - 30 \times 13 \times 9 \) km, (253) Mathilde \( - 66 \times 48 \times 46 \) km and (433) Eros \( - 33 \times 13 \times 13 \) km.

Itokawa’s global shape has been compared with a sea otter. Three regions were distinguished: a small lobe called the “head”, a large lobe called the “body” or “bottom”, united by a finer zone called the “neck” (Fujitani et al., 2006) (see Fig. 1). On the surface we can distinguish two distinct types of terrain: a rough terrain that covers most of the surface that includes numerous large boulders, and a few patches of a flat and smooth terrain covered by regolith. The rough terrains are found in the “head” and the “bottom”, while the smooth terrain is found in the “neck”.

By analysing the Itokawa’s rotational lightcurves of over a decade, Lowry et al. (2014) computed the YORP-induced radiative torques and they found that Itokawa might have a centre-of-mass which is displaced from the geometric centre-of-figure. This result was interpreted using different models of an inhomogeneous interior: Model (1) an object composed of two separate bodies with very different bulk densities, or Model (2) an object with a compressed neck region of higher density located between the body and head.

The total number of blocks larger than 5 m is roughly 500 (Saito et al., 2006). This leads to a number surface density of blocks larger than 5 m of \( 1.3 \times 10^8 \) km\(^{-2}\), more than an order of magnitude larger than the number observed in the Near-Earth 5-type asteroid Eros. The cumulative boulder size distribution per unit area of Itokawa was first computed by Saito et al. (2006), they obtained a power-index of \(-2.8\) down to a boulder’s diameter of approximately 5 m on the entire surface. A more detail analysis of the size distribution based on high-resolution images was done by Michikami et al. (2008). Nonetheless, the analysis was limited to boulders larger than 5 m. They obtained a mean index on the entire surface of \(-3.1 \pm 0.1\). But they made a separate analysis for the East and West sides and the head and body portions, obtaining a range of values from \(-2.8\) to \(-3.2\). The boulder’s size distribution was further studied by Mazzouei et al. (2014). By applying a different measuring technique and a better method to compute the cumulative size distribution, they obtained a mean index on the entire surface of \(-3.3 \pm 0.1\), and index of \(-3.1 \pm 0.4\) and \(-3.6 \pm 0.2\) for the head and body, respectively. A drawback of all these previous studies was the limited range of boulder sizes to compute the distribution: from 5 to 30 m, less than one decade.

In this work we revisit the problem of the distribution of boulders on the surface of Itokawa, but we extend the analysis to a larger set of regions and to boulders of smaller sizes. In order to analyse the distribution we require: to define a shape model (Section 2); to obtain the viewing conditions of the images (Section 3); and to compute the total potential and gravity on the surface (Section 4). The distribution of boulders on the surface is presented in Section 5. In the appendix we raise the question whether the distribution of boulders on the surface represents the distribution of the interior. The results are analysed in Section 6 and the conclusions are presented in Section 7.

### 2. The shape model

The shape model is constructed by applying multi-image photoclino-ometry on the set of ~600 science images provided by the AMICA instrument on board the Hayabusa spacecraft (Gaskell et al., 2008a). The global topography models (GTM) are derived from the implicitly connected quadrilateral format. The construction of a GTM begins with a low-resolution reference shape such as a tri-axial ellipsoid. The shape model comprises triangular plates (facets) and vertices that are specified in a body-fixed coordinate system. The vertices are represented by three-vectors and the facets are represented by a set of 3 vertices’ indexes. Gaskell et al. (2008b) provide several shape models with different number of vertices and facets. The number of vertices are scaled with the following equation: \( N_{\text{vertices}} = 6 (q + 1)^2 \), where \( q = [64, 128, 256, 512] \). In this work we will generally use the model with \( q = 128 \), i.e. 99,846 vertices and 196,608 facets.

The Itokawa fixed frame is defined in a way that the largest diameter corresponds with the \( x \)-axis of the asteroid ellipsoid and the positive direction goes towards the “head” region. The \( z \)-axis corresponds with the spin vector and the positive \( y \)-axis can be derived from the latter two.

Using the GTMs, Gaskell et al. (2008a) made a new estimate of Itokawa’s surface area of 4.0403 \( \times 10^3 \) m\(^2\) and of the volume of 1.773 \( \times 10^3 \) m\(^3\).

### 3. Viewing the images

To compare images taken with the AMICA camera on board Hayabusa and the shape model, we required information about the viewing configuration of the camera relative to the asteroid. To compute the viewing parameters we used the information system named SPICE provided by NASA.

SPICE (Acton et al., 2011) is a set of software routines (the SPICE toolkit) and a suite of data formats that help a scientist use ancillary data to plan scientific observations from a space vehicle and to analyse the science data gathered from those observations. It is comprised of a set of packages called kernels that have critical information about the spacecraft, instruments, and all objects and parameters involved in the mission. Some kernels specific to the mission can be downloaded from the NASA-PDS website. These kernels tell us for example the pointing direction of the AMICA camera relative to a fixed coordinate frame in the spacecraft; these are called Frame Kernels.

With the support of J.L. Vázquez García, we made use of the SPICE library and kernels to compute the camera position, the camera target and the camera up vector from the UTC time of the exposure that appears in the image header. In Fig. 1 we present an image of Itokawa and a model with 49,152 triangular facets (\( q = 64 \)).

Recently, Barnouin and Kahn (2012) have provided a data set of images taken by the AMICA camera, supplemented with information for each pixel. The data set is named: Hayabusa AMICA Images with Geometry Backplanes (hereafter HAIGB). This data set includes the Derived Data Record (HAIGB-DDR) for 1339 images in FITS format. Each of the HAIGB-DDR files is essentially a 16-layered image cube. The HAIGB-DDR contains geometric

![Fig. 1. Overlapping of triangular facets on an image taken with the AMICA camera on-board of Hayabusa.](image-url)
information for each pixel of these images. The first layer or band of this cube has the pixel values of the original fits images delivered to the PDS by the Hayabusa team. The remaining layers include the location of each pixel on the surface of the asteroid, the solar incidence angle, emission angle and phase angle at each pixel when the image were acquired, and data on the local slope relative to gravity, the acceleration due to gravity and the elevation relative to a reference gravitational potential at each pixel, assuming a constant density asteroid.

4. The total potential and surface gravity

The total potential \( \Phi_{\text{total}} \) is the sum of the gravitational potential \( \Phi_{\text{grav}} \) plus the rotational centrifugal potential \( \Phi_{\text{rot}} \):

\[
\Phi_{\text{total}} = \Phi_{\text{grav}} + \Phi_{\text{rot}},
\]

(1)

the unit of the potentials are J/kg.

Let's consider a body with a volume \( V \) and internal density \( \rho \), which depends on the position \( \mathbf{r} \). The gravitation potential at a position \( \mathbf{r} \) is given by:

\[
\Phi_{\text{grav}}(\mathbf{r}) = -\frac{G}{V} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, d^3\mathbf{r}'.
\]

(2)

A solid body rotating with an angular velocity \( \mathbf{\omega} \) has a rotational centrifugal potential \( \Phi_{\text{rot}} \) given by

\[
\Phi_{\text{rot}}(\mathbf{r}) = -\frac{1}{2} \left( \mathbf{\omega} \times \mathbf{r} \right)^2.
\]

(3)

The surface gravity vector \( \mathbf{g} \) is calculated by

\[
\mathbf{g} = -\nabla \Phi_{\text{total}}.
\]

(4)

The fifteenth and sixteenth layer of the HAIBG-DDR images provide the modulus of the surface gravity and average gravitational potential at each pixel on the surface of the asteroid, respectively. These values were calculated using the method described in Cheng et al. (2002) using a constant density for Itokawa of \( 1.95 \text{ g/cm}^3 \) and a rotation rate of \( 1.44 \times 10^{-4} \text{ rad/s} \). This density and rotation rate the surface gravity and gravitational potential on the surface can be computed via integration (Cheng et al., 2002; Barnouin-Jha et al., 2008, see e.g.). The computation method has its limitation because an interior constant density is assumed and the surface is approximated by a set of facets or plates. Nevertheless, the plate models used for the purpose of determining the DDR backplanes were the closest in resolution to the pixel scale of a particular AMICA image Barnouin and Kahn (2012), and therefore the differences should be small.

In Figs. 2 and 3 we select a few grey-scale images to show different sides of Itokawa and we overlap colour coded plots of the total potential and the surface gravity given by the Barnouin and Kahn (2012) data set, respectively. The total potential is in the range \(-0.01677 \text{ J/kg} \) in Muses-C to \(-0.01129 \text{ J/kg} \) in the “head”. The surface gravity is in the range \( 5.68 \times 10^{-6} \text{ m/s}^2 \) in the “head” to \( 9.04 \times 10^{-6} \text{ m/s}^2 \) in Muses-C. Boulders of sizes in the range tenths to tens of meters can be observed on the surface. As already noted by Fujiwara et al. (2006), there is a correlation between the size of the boulders and the potential: large boulders are concentrated in the regions of higher potential (the reddish areas in the “head” and the “bottom”), and smaller boulders are located in regions of lower potential (the bluish area in the “neck”). Similarly the highlands in the “body” with medium and large-size boulders have intermediate values of the potential (whitish areas). Similar considerations can be drawn between the surface gravity and the distribution of boulders on the surface: large boulders are located in the regions of low surface gravity, and small ones in the regions of high surface gravity. This qualitative conclusion is clearly shown in the previous overlapped plots.

5. The distribution of boulders on the surface

5.1. Size distribution

Particles (grains) are classified according to their sizes. In Geology, particle rocks larger than 256 mm are defined as boulders after Wentworth (1922).

In order to make a quantitative analysis of the correlation between the potential and the boulder sizes, we extend the

![Fig. 2. Overlapping of the total potential on images taken with the AMICA camera. Images used in this composite are: (a) ST_2417520833_v – East, (b) ST_2482160259_v – “head”, (c) ST_2421011334_v – West, and (d) ST_2424157005_v – “bottom”.](image-url)
boulder counting done by Michikami et al. (2008). We select several images in different regions on the surface of Itokawa and compute the size distribution in each of them. Images were selected from: the “head”, the “bottom” (“Arcoona Regio”), “body” highlands (“Ohsumi Regio”), North Pole (“Sagamihara Regio”), Transition zone (close to “Komaba Crater”) and several areas in “Muses-C Regio”. The regions used for boulder counting are highlighted in Fig. 4 and listed in Table 1. In Fig. 5, these regions are located in the same maps of the total potential shown in Fig. 2.

In each regions we have done two separate analysis:

- **Boulder counting** – The images are loaded in SAOImage DS9. The images are visually inspected and the boulders are recognised by eye. The “Region” tool in DS9 is used to overlap an ellipse on top of each boulder. Each ellipse can be rotated to fit the orientation of the boulder. An ASCII table is produced with information of the location of the centre of the ellipse, the semi-major and semi-minor axes (in pixel units) and the orientation angle. A couple of examples of the identification of boulders are presented in Fig. 6. The numbers of boulders counted in each region are listed in Table 1.

- **Region’s parameters** – We require information about several parameters of each region, e.g.: location, area, image scale, emission angle, mean value of the total potential and surface gravity. This information can be obtained from the shape model shown in Section 2; the viewing conditions obtained from SPICE as described in Section 3; and the potential and surface gravity computed as in Section 4. Nonetheless, some of the studied regions are included in the data set provided by Barnouin and Kahn (2012). We use the HAIGB-DDR files based on the Gaskell et al. (2008a) shape model.

For those images not included in the above mentioned data set, those identified with the letter “L” in the Type column of Table 1, we extract the distance of the Hayabusa spacecraft to the surface from the data set provided by Mukai et al. (2012) of the LIDAR instrument. From the distance and the information about the AMICA camera provided by Ishiguro et al. (2010) (pixel size: 12 μm; focal distance: 0.1208 m), it is possible to compute the image scale (scale = distance × pixel size/focal distance). The surface area is then calculated as the area in pixels times the square of the image scale. These regions are then identified in wider images in order to get an estimate of the location, total potential and surface gravity. All the region’s parameters are listed in Table 1.

To compute the size distribution, we do the following steps in each region:

- Compute the equivalent radius (in m) of the boulders (assuming a circle of equal area to the ellipse), as

  \[ r = \sqrt{\frac{ab \cdot \text{hesc}_i \cdot \text{vesc}_i}{\cos(\epsilon_i)}} \]  

  where hesc and vesc are the horizontal and vertical scale at the centre of the ellipse, respectively; and \( \epsilon \) is the emission angle at this point.

- Sort the boulders in decreasing size and assign an increasing index (N).
With these data we compute the cumulative number size distribution in each region (Fig. 7). Note that in most of the regions the cumulative number cover over 2 up to 3 order of magnitudes, totalising over 3700 boulders counted on the surface. Since the counts were done in regions of different areas, in order to compare the distributions, we transform the cumulative number into cumulative number per unit area. The following steps are performed:

- Compute the cumulative number per unit area \( \Omega = N/\text{Area} \) [m\(^{-2}\)].
- Compute the parameters of the cumulative number per unit area function (CNF) as described below.

The cumulative number densities for the different regions are presented in Figs. 8 and 9. Note that there are some regions in Fig. 5 that overlap but the images have different pixel scales; like the regions 6 and 7, 9–13, and 14 and 15. In Fig. 9, these regions are plotted with symbols with the same face colour, but different shapes. By combining data from images of different pixel scale of similar regions, we have been able to extend the size distribution down to sub-m size boulders; while in Saito et al. (2006) and Michikami et al. (2008), the smallest boulders in their size distributions were \( \sim 5 \) m.

The boulders' size distribution are usually fitted to a power law probability density \( p \) of the type
Table 1
List of parameters of the regions where boulder counting was performed.

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Region</th>
<th>Image</th>
<th>Lat (deg)</th>
<th>Lon (deg)</th>
<th>Area (px)</th>
<th>Area (m²)</th>
<th>Scale (m/px)</th>
<th>Potential (J/kg)</th>
<th>Gravity $10^{-5}$ (m/s²)</th>
<th>Type (’)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>“head” 1</td>
<td>ST_2482160259_v</td>
<td>-4</td>
<td>359</td>
<td>115922</td>
<td>36982.3</td>
<td>0.4348</td>
<td>-0.0124 ± 0.0006</td>
<td>6.777</td>
<td>G</td>
</tr>
<tr>
<td>2</td>
<td>“bottom” 1</td>
<td>ST_249817622_v</td>
<td>2</td>
<td>178</td>
<td>69312</td>
<td>13157.6</td>
<td>0.3785</td>
<td>-0.0139 ± 0.0004</td>
<td>7.915</td>
<td>G</td>
</tr>
<tr>
<td>3</td>
<td>“bottom” 2</td>
<td>ST_2516321279_v</td>
<td>-4</td>
<td>172</td>
<td>524288</td>
<td>8561.5</td>
<td>0.1168</td>
<td>-0.0136 ± 0.0003</td>
<td>7.761</td>
<td>G</td>
</tr>
<tr>
<td>4</td>
<td>“body” highlands West 1</td>
<td>ST_2494934387_v</td>
<td>-6</td>
<td>271</td>
<td>225766</td>
<td>34336.6</td>
<td>0.3174</td>
<td>-0.0153 ± 0.0005</td>
<td>8.400</td>
<td>G</td>
</tr>
<tr>
<td>5</td>
<td>“body” highlands West 2</td>
<td>ST_2516129281_v</td>
<td>-4</td>
<td>229</td>
<td>454326</td>
<td>26534.5</td>
<td>0.1848</td>
<td>-0.0149 ± 0.0005</td>
<td>8.400</td>
<td>G</td>
</tr>
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<td>6</td>
<td>“body” highlands East 1</td>
<td>ST_2506733028_v</td>
<td>1</td>
<td>142</td>
<td>139350</td>
<td>48347.4</td>
<td>0.4249</td>
<td>-0.0145 ± 0.0008</td>
<td>8.146</td>
<td>G</td>
</tr>
<tr>
<td>7</td>
<td>“body” highlands East 2</td>
<td>ST_2539451160_v</td>
<td>3</td>
<td>131</td>
<td>1021952</td>
<td>61.8</td>
<td>0.0078</td>
<td>-0.0144 ± 0.0001</td>
<td>8.011</td>
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<tr>
<td>8</td>
<td>Sagamihara</td>
<td>ST_2486640220_b</td>
<td>30</td>
<td>248</td>
<td>320865</td>
<td>10403.5</td>
<td>0.4801</td>
<td>-0.0160 ± 0.0001</td>
<td>8.940</td>
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<tr>
<td>9</td>
<td>Transition zone 1</td>
<td>ST_2532629277_v</td>
<td>-18</td>
<td>91</td>
<td>453118</td>
<td>3008.6</td>
<td>0.0630</td>
<td>-0.0162 ± 0.0002</td>
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<tr>
<td>10</td>
<td>Transition zone 2</td>
<td>ST_2539437177_v</td>
<td>-2</td>
<td>105</td>
<td>1021952</td>
<td>127.6</td>
<td>0.0112</td>
<td>-0.0156 ± 0.0001</td>
<td>8.450</td>
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<tr>
<td>11</td>
<td>Transition zone 3</td>
<td>ST_2539429953_v</td>
<td>-7</td>
<td>90</td>
<td>1021952</td>
<td>253.3</td>
<td>0.0158</td>
<td>-0.0161 ± 0.0001</td>
<td>8.515</td>
<td>L</td>
</tr>
<tr>
<td>12</td>
<td>Transition zone 4</td>
<td>ST_2539423137_v</td>
<td>-8</td>
<td>71</td>
<td>1021952</td>
<td>484.5</td>
<td>0.0218</td>
<td>-0.0164 ± 0.0001</td>
<td>8.438</td>
<td>L</td>
</tr>
<tr>
<td>13</td>
<td>Muses-C edge</td>
<td>ST_2532629277_v</td>
<td>-35</td>
<td>62</td>
<td>292240</td>
<td>2720.9</td>
<td>0.0673</td>
<td>-0.0167 ± 0.0001</td>
<td>8.490</td>
<td>G</td>
</tr>
<tr>
<td>14</td>
<td>Muses-C 1</td>
<td>ST_2495806075_v</td>
<td>-33</td>
<td>56</td>
<td>24807</td>
<td>4278.3</td>
<td>0.3269</td>
<td>-0.0166 ± 0.0001</td>
<td>8.425</td>
<td>G</td>
</tr>
<tr>
<td>15</td>
<td>Muses-C 2</td>
<td>ST_25635111720_v</td>
<td>-16</td>
<td>46</td>
<td>1021952</td>
<td>65.8</td>
<td>0.0080</td>
<td>-0.0165 ± 0.0000</td>
<td>8.181</td>
<td>L</td>
</tr>
</tbody>
</table>

(*) - Type of method to compute the area and the scale parameters: G-based on the Barnouin and Kahn (2012) Geometry Backplanes; L-based on the distance to the surface given by the LIDAR instrument (Mukai et al., 2012). Images ST_2539429953_v and ST_25635111720_v were taken from distances of 159.2 and 80.85 m, respectively. These regions are identified in images with a larger view, like ST_2482160259_v and ST_2495806075_v, respectively, in order to get the estimates of the potential and surface gravity.
where $\alpha$ is the constant exponential parameter known as the exponent and $C$ a normalisation constant and $D$ is the boulder diameter. The power law represents boulders down to some minimum boulder diameter ($D_{\text{min}}$), giving the normalisation constant $C = (\alpha - 1)D_{\text{min}}^{-\alpha}$, provided $\alpha > 1$. The cumulative distribution function ($P$) is then

$$P(D) = \left(\frac{D}{D_{\text{min}}}\right)^{-\alpha + 1}.$$  \hspace{1cm} (8)

The cumulative number per unit area function is then calculated as:

$$\text{CNF} = P(D)/\text{Area}.$$  

The exponent of the distributions as well as the minimum diameters are estimated with a method which combines a
maximum-likelihood fitting method with goodness-of-fit tests based on the Kolmogorov–Smirnov statistic, as described by Clauset et al. (2009). The maximum-likelihood estimator of the exponent $x$ is given by:

$$
\hat{x} = 1 + n \left[ \frac{\sum_{i=1}^{n} \ln \left( \frac{D_i}{D_{\text{min}}} \right)}{n} \right]^{-1},
$$

where $D_i$, $i = 1, \ldots, n$, are the observed values of the diameters, such that $D_i \geq D_{\text{min}}$, and $n$ is the total number of points. To estimate the minimum diameter, we compute the Kolmogorov–Smirnov statistic for different values of $D_{\text{min}}$, which is simply the maximum distance between the CNFs of the data and the fitted model; the estimate $D_{\text{min}}$ is the value of $D_{\text{min}}$ that minimizes this distance.

The exponent of the CNF $(x - 1)$ and the minimum diameter $(D_{\text{min}})$ are listed in Table 2, as well as the numbers of objects $(N_{\text{obj}})$ up to $D_{\text{min}}$. The diameters $(D_{\text{com}})$ corresponding to a cumulative number per unit area $\Omega = 0.01\text{ m}^{-2}$ are also listed in this Table, as well as the cumulative number per unit area $\Omega_{\text{bf}}$ for a given value of $D = 2\text{ m}$. The uncertainty in $x$ is computed as:

$$
\sigma_x = \frac{x - 1}{\sqrt{n}} + O(1/n).
$$

The uncertainty in $D_{\text{com}}$ is computed as the intersection of the power laws with exponents $x - 1 \pm \sigma_x$ with the cumulative number per unit area $\Omega = 0.01\text{ m}^{-2}$. A similar procedure is applied for the computation of the uncertainty in cumulative number per unit area $\Omega_{\text{bf}}$ for a given value of $D = 2\text{ m}$. The uncertainty in $x$, $D_{\text{com}}$ and $\Omega_{\text{bf}}$ are shown in the error bars in the later Fig. 11.

In addition to the estimate of the exponents and the minimum diameters of the CNFs, we applied a goodness-of-fit tests to the data sets. Following Clauset et al. (2009), we generate synthetic data sets with the given values of exponents and minimum diameters for each CNF, and we compare the Kolmogorov–Smirnov statistic of the best-fit power law and the synthetic data set. The $p$-value is defined to be the fraction of the synthetic distances that are larger than the empirical distance. All the fitted power-law shown $p$-values significantly larger than 1, which means that the differences between the empirical data and the model can be attributed to statistical fluctuations, and the power-laws are a good fit to all the data sets.

There are some regions where the cumulative number per unit area $\Omega$ should be fitted with a broken power law with a high exponent for the large diameters (“max exponent”) and a low exponent for the small ones (“second exponent”). This occurs in regions 4, 5, 9, and 11, as shown in Fig. 7. Note that the diameters limiting the two power laws are much larger than the cut-off diameter due to completeness, therefore the broken power law seems to be a real effect and not an artifact as a consequence of an incompleteness bias. In these cases of broken power laws, we compute the exponent with the method described above for the large diameters, while for the small diameters we did a linear fit from the $D_{\text{min}}$ obtained in the large diameter fit, down to the cut-off diameter (see e.g. the two curves in Fig. 8b). The second exponent as well as the other parameters of the fits are also listed in Table 2.

Some overlapping regions with different pixel scales present different exponents, like the cases of regions 6 and 7 (Fig. 8c), and 14 and 15 (Fig. 8e). These cases also correspond to regions with cumulative number densities that should be fitted with a broken power law, but the diameter at the breaking point is in between the smaller values of the regions with the larger pixel scale (regions 6 and 14) and the larger values of the regions with the smaller pixel scale (regions 7 and 15, respectively).

On the other hand, the regions 9–12 in the “Transition zone” have overlapping CNF (Fig. 8d). In region 9 and 11 we are able to fit broken power laws, while regions 10 and 12 have a CNF with an exponent similar to the second exponent of the two previous regions.

Michikami et al. (2008) computed the size distribution down to boulders of ~5 m in 4 wide regions of the surface of Itokawa. They obtain the following values of the exponents: “head” – $x = -2.8 \pm 0.1$, “bottom” – $x = -3.2 \pm 0.1$, West – $x = -2.9 \pm 0.1$,

![Fig. 9. Cumulative (per unit area) distribution of boulders on different regions over the surface. The lines are colour coded according to the groups described in Fig. 8. In the upper right corner we draw blue lines with slopes of $-1$, $-2$, $-3$ and $-4$. The horizontal black full-line corresponds to a cumulative number per unit area $\Omega = 0.01\text{ m}^{-2}$. The vertical black dashed-line corresponds to a boulder’s diameter $D = 2\text{ m}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image-url)
East - $\alpha = -3.2 \pm 0.1$. Meanwhile, Mazrouei et al. (2014) obtained an exponent of $\alpha = -3.5 \pm 0.1$ for all the blocks larger than 6 m; separating into the “head” and the “body” (“bottom”), the exponents are $\alpha = -3.1 \pm 0.4$ and $\alpha = -3.6 \pm 0.2$, respectively. The exponents obtained in our analysis cover a wider range of values, in particular in the cases of broken power laws. We note a few differences between our method to present the size distribution and to compute the exponent respect to the previous works. Michikami et al. (2008) computed the size distribution by binning the data, which it is not recommended as already pointed out by Mazrouei et al. (2014) (Clauset et al., 2009, see also). Either Michikami et al. (2008) and Mazrouei et al. (2014) computed the exponents of the cumulative size distributions by applying a linear fit in the log–log plot up to a completeness diameter determined by eye. Clauset et al. (2009) had shown that this least-squares fitting method can produce substantially inaccurate estimates of parameters for power-law distributions. In addition, our estimates of the exponents were done by using a wider range of diameters.

Taking into account these caveats, we proceed to compare our results with the previous works. In the “head” we obtain a value of $\alpha = -2.98 \pm 0.36$, while in the “bottom” (“body” in their plot) we have a couple of measurements: $\alpha = -2.68 \pm 0.38$ and $\alpha = -3.22 \pm 0.51$. These values are compatible with Michikami et al. (2008) estimates, but they are somewhat smaller (in absolute values) respect to Mazrouei et al. (2014). In the West and East side we fit broken power laws; the exponents of Michikami et al. (2008) are in between our estimates of the first and second exponents. Looking at their Fig. 4, where they plot the cumulative number per unit area for the West and East sides, we note that also in their case a broken power law could be observed, although they only fit a single power law.

### 5.2. Shape distribution

Since we have fitted an ellipse to each boulder, we have information about the ellipticity of the boulders. The ellipticity is defined as $e = \sqrt{1 - \frac{b^2}{a^2}}$. Round boulders have ellipticities close to 0, while elongated boulders have values close to 1.

The differential and cumulative distributions of the boulder’s ellipticities in the different regions are presented in Fig. 10a and b, respectively. The regions are listed in decreasing values of the gravitational potential. We choose regions with a similar boulder’s size range; we explicitly exclude regions with close-up images and those with few boulders. Note that in the cumulative distribution (Fig. 10b) the curves of the lower potential are generally shifted to larger ellipticities. This result is discussed in Section 6.2.

### 6. Analysis and results

#### 6.1. Size segregation

We compare the CNFs of the different regions in relation to the value of the total potential and the surface gravity. In Fig. 11, we show the maximum exponent of the CNFs ($\alpha - 1$) in each region (a), and the second exponent in the case of broken power laws (b) versus the total potential. In Fig. 11c and d we present the diameters ($D_{com}$) corresponding to a cumulative number per unit area $\Omega = 0.01$ m$^{-2}$ and the cumulative number per unit area $\Omega_T$ for a given value of $D = 2$ m, respectively, versus the total potential. $D_{com}$ and $\Omega_T$ are used to compare the location of the CNFs curves in the $\Omega - D$ phase space.

We apply a linear fit to the data in each of the plots of Fig. 11 just to highlight the trends in the dataset. The fits were done with a sigma-clipping method; i.e.: after the linear fit, the data points that are further than $3\sigma$ from the predicted value are discarded, and a new fit is computed until the process converges. For the fits presented in the plots of Fig. 11 it was not necessary to discard any data point.

As shown in Fig. 11a, there is a slight trend of steeper CNF (larger exponent) with lower potentials. The second exponent presented in Fig. 11b shows a more pronounced trend. In Fig. 11c and d, we observe a decrease in the corresponding diameter for a given cumulative number per unit area, or a decrease in the cumulative number per unit area a given diameter, which it can be interpreted as a shift of the CNFs towards smaller boulders for lower values of the total potential. In addition, the existence of broken power laws could be interpreted in the same framework. The CNFs for the high potential regions have single power laws (at least in the range of boulder sizes analysed in our counts) with small exponents. For intermediate potentials, there is an under abundance of large boulders and an overabundance of intermediate size ones. This effect produces a broken power law, with a large exponent for the large sizes and a lower exponent for the small sizes. For low potentials, there are fewer large boulders and more small ones. Therefore, the diameters where the power laws break are shifted towards smaller sizes for lower potentials (Fig. 11e).

These results support global size segregation of boulders in Itokawa, and are in agreement with the preliminary observations presented in Section 4.

Since the size distributions observed on the surface represent the situation in the interior below, as it was shown in the appendix, there appears to be a global relocation of boulders, with large ones going into the high potential regions and small ones into the low potential ones. Size segregation is a well-known phenomena in granular physics (Rosato et al., 1987; Knight et al., 1993; Rosato et al., 1997).
It is also known as the Brazil Nut Effect (BNE), because it can be easily seen when one mixes nuts of different sizes in a can; the large Brazil nuts rise to the top of the can. Unless there is a large difference in the density of the grains, a mixture of different particles will segregate by size when shaken: large particles rise to the top occupying the region of higher gravitational potential; while small ones go to the regions of lower potential, i.e., the bottom.

As it is speculated in Miyamoto et al. (2007), the size sorting observed in Itokawa could be due to the seismic shaking produced by repetitious impacts. Nevertheless, they have not performed any experiments to test this.

Tancredi et al. (2012) performed numerical simulations based on the Discrete Element Methods (DEM) of a set of particles in different gravity environments under the action of several shaking processes. They have shown that the Brazil nut effect is present even in low-gravity environments like the surface of Itokawa. They simulate a 3D box filled with many small spherical particles and a large one undergoing repeated displacements of the floor under different gravity regimes. The numerical experiments try to reproduce the behaviour of boulders close to the surface under the passage of a seismic shock coming from below. They observed that the Brazil nut effect operates in all gravity regimes (including Itokawa), on different timescale and for threshold displacement velocities that decrease as surface gravity decreases.

Nevertheless, these tests only show that size segregation could occur on the surface of a low-gravity object. It is still necessary to prove that global relocation of boulders correlated with the gravity field could occur. Numerical simulations of this process are in development, and the results will be presented elsewhere.

The global relocation of boulders could have consequences for the mass and density distribution in the body’s interior. If the small particles concentrate in the region with the lowest potential close to the centre-of-mass, and they tend to have a higher degree of compacting, the body could acquire a mass distribution similar to the second model proposed by Lowry et al. (2014) to explain the displacement of the centre-of-mass, the model with a compressed “neck” region of higher density located between the...
“body” and “head” (see Section 1). This hypothesis should be tested with numerical simulations of the global relocation of boulders, like the ones we have in progress.

6.2. Shape segregation

Shape segregation was observed in numerical simulations of spherocylinders (Abreu et al., 2003) as well as particles with more complex shapes (Roskilly et al., 2010). For these simulations the authors did not use DEM, but rather a Monte Carlo method, which is another modelling technique that has been applied to complex materials including particulates. In Abreu et al. (2003) the spherocylinders were characterised by the aspect ratio \( \phi \) defined as the ratio between the length of the cylindrical portion and the diameter of the hemispherical part. For a binary mixture of spherocylinders of very different aspect ratios, they found a segregation of round particles to the bottom and long spherocylinders to the top.

An ellipse can be approximated by a 2D spherocylinder of length equal to two times the focal length and a diameter equal to two times the peripaxis distance. The aspect ratio is then calculated as: \( \phi = 1/(1/e - 1) \).

In Roskilly et al. (2010), the particulates were characterised by the radius of gyration. The radius of gyration of a 3D object is calculated as \( R = \sqrt{I/M} \), where \( I \) is the moment of inertia and \( M \) the mass of the object, while for a 2D object is \( R = \sqrt{I/A} \), where \( A \) is the area. An average over the radii of gyration with respect to the principal axis was considered by the authors as an indication of the ‘effective size’ of the objects. Roskilly et al. (2010) concluded that: “…when a particulate system containing particles of exactly the same size, but different shapes, is subjected to large amplitude low frequency shaking, the particles with the largest ‘effective size’, as measured by the radius of gyration, move to the top.” They concluded that the process of shape segregation is similar to the size segregation discussed above; they said: “it appears that segregation may occur by voids opening below larger objects that can only be filled with smaller particles.”

The radii of gyration of an ellipse are: \( R = (a/2, b/2) \); where \( a \) and \( b \) are the semiaxes of the ellipse. The ratio of average radius of gyration of an ellipse respect to a circle of the same area can be expressed in term of the eccentricity of the ellipse as:

\[
R_{\text{relative}} = (a + b)/2\sqrt{ab} = (1 + \sqrt{1 - \epsilon^2})/2(1 - \epsilon^2)^{1/4}.
\]

Nevertheless, results presented in the thesis of Ramaioli (2008) are in contradiction to the results presented by Abreu et al. (2003) and Roskilly et al. (2010). While the later authors used Monte Carlo simulations, Ramaioli (2008) performed simulations using DEM, as well as laboratory experiments with glass spherocylinders. Ramaioli (2008) concluded that short spherocylinders tend to migrate to the top of long spherocylinders; a trend opposite to the other authors. Further simulations and experimental studies should be done to settle this problem.

What do we observe in our dataset? The differential and cumulative distributions of ellipticities were shown in Fig. 10a and b, respectively. The regions are listed in decreasing value of the potential. Note that in the cumulative distribution of ellipticities presented in Fig. 10b, the curves corresponding to regions of high potential are above to those of low potential, particularly in the regions of large ellipticities. In Fig. 12a and b we present the medians and the upper quartiles (75%) of each distribution of ellipticities as a function of the corresponding total potential, and apply a linear fit to the data. A trend of decreasing median and upper quartiles ellipticities with increasing total potential is observed. Similar trends are observed in the distribution of aspect ratios and radii of gyration. Round boulders are located in the regions of higher potential, while more elongated objects are in the regions of lower potential. This is shape segregation similar to that observed by Ramaioli (2008), in opposition to the trend expected by the results by Abreu et al. (2003) and Roskilly et al. (2010).

7. Conclusions

Our main conclusions are summarised as follows:

- There is a correlation between the size of the boulders and the potential: large boulders are concentrated in the regions of higher potential, and small boulders are located in regions of lower potential, as shown in the figures where we overlap a grey-scale of the surface of Itokawa and a colour coded plots of the total potential.
- The cumulative size distribution on different regions of the surface of Itokawa was computed and fit to a power-law function. The exponents and the minimum diameters corresponding to a given cumulative number per unit area were computed for each region.
- Based on modelling the distribution of boulders, we found that the size distribution computed from an analysis of the boulders on the surface is a good representation of the size distribution in the interior just below.
- There is a slight trend of steeper cumulative number per unit area function (larger exponent) with lower potentials; as well as a shift of the cumulative number per unit area function to smaller sizes for lower potentials and higher surface gravities.
- Regions with intermediate potentials present broken power laws, which correspond to an overabundance of intermediate size boulders. The diameter where the power is discontinuous is shifted towards smaller sizes for lower potentials.
• These results support the hypothesis that the size segregation is a global process; there is a global relocation of boulders, with large boulders going into the high potential regions and small boulders into the low potential ones.

• The size sorting could be due to the seismic shaking produced by repetitious impacts, as it was shown to operate in the low-gravity environment of Itokawa by Tancredi et al. (2012).

• Shape segregation is observed on the distribution of boulders: more rounded boulders are found in the regions of high potential, while more elongated objects are more frequent in regions of low potential. This result is in accordance to the DEM simulation and laboratory experiments preliminary presented by Ramaioloi (2008); but in contradiction to the numerical simulations based on Monte Carlo techniques presented by Abreu et al. (2003) and Roskilly et al. (2010).

Acknowledgments

We would like to thank Edward Stokan for a careful reading of the text and many linguistic suggestions. We also thanks Sofia Favre for her participation in an early stage of the work. The support by Makoto Yoshikawa and Jose Luis Vázquez García in dealing with the Itokawa data set and SPICE, respectively, was greatly appreciated. Comments and suggestions by the anonymous reviewers were very useful to improve the manuscript.

Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.icarus.2014.10.011.

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